

## INTEGRATING RATIONAL FUNCTIONS OF THE FORM $\frac{f'(x)}{f(x)}$

Rule:  $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$  since  $\frac{d}{dx} \ln[f(x)] = \frac{f'(x)}{f(x)}$ .

Examples: Integrate these functions:

a)  $\int \frac{10x}{5x^2} dx$

Since  $\frac{d}{dx} 5x^2 = 10x$ , then  $\int \frac{10x}{5x^2} dx$  fits the form  $\int \frac{f'(x)}{f(x)} dx$  where  $f(x) = 5x^2$ .

Therefore  $\int \frac{10x}{5x^2} dx = \ln|f(x)| + c = \ln 5x^2 + c$ .

b)  $\int \frac{18x}{3x^2+5} dx$

Since  $\frac{d}{dx} 3x^2 + 5 = 6x$ ,  $\int \frac{18x}{3x^2+5} dx$  can be made to fit the form  $\int \frac{f'(x)}{f(x)} dx$  by adding a multiplier:  $\int \frac{18x}{3x^2+5} dx = \int \frac{3 \times 6x}{3x^2+5} dx = 3 \int \frac{6x}{3x^2+5} dx$ .

The multiplier is 3, i.e. the integral  $\int \frac{18x}{3x^2+5} dx$  can be written as the equivalent  $3 \int \frac{6x}{3x^2+5} dx$ , which is in the form  $3 \int \frac{f'(x)}{f(x)} dx$ .

So  $\int \frac{18x}{3x^2+5} dx = 3 \int \frac{6x}{3x^2+5} dx = 3 \ln|3x^2 + 5| + c$ .

c)  $\int \frac{4x e^{x^2}}{3e^{x^2}+5} dx$

$\frac{d}{dx} 3e^{x^2} + 5 = 2x \cdot 3e^{x^2} = 6xe^{x^2}$  using the rule  $\frac{d}{dx} e^{f(x)} = f'(x)e^{f(x)}$ .

$$\int \frac{4x e^{x^2}}{3e^{x^2}+5} dx = 4 \int \frac{x e^{x^2}}{3e^{x^2}+5} dx = \frac{4}{6} \int \frac{6x e^{x^2}}{3e^{x^2}+5} dx = \frac{2}{3} \int \frac{6x e^{x^2}}{3e^{x^2}+5} dx,$$

which is in the form  $\frac{2}{3} \int \frac{f'(x)}{f(x)} dx$ . Therefore  $\frac{2}{3} \int \frac{6x e^{x^2}}{3e^{x^2}+5} dx = \frac{2}{3} \ln|3e^{x^2} + 5| + c$